

### Problem # 314

Let  $x_1$ ,  $x_2$ , and  $x_3$  be the roots of the polynomial  $x^3 - 4x^2 + 2x - 3$ .

Find the value of  $\frac{x_3}{x_1 x_2} + \frac{x_1}{x_2 x_3} + \frac{x_2}{x_3 x_1}$ .

**Solution:**

*Answer:* 4

*Proof.* Notice that

$$(x - x_1)(x - x_2)(x - x_3) = x^3 - (x_1 + x_2 + x_3)x^2 + (x_1x_2 + x_2x_3 + x_1x_3)x - x_1x_2x_3,$$

and

$$(x_1 + x_2 + x_3) = 4, \quad (x_1x_2 + x_2x_3 + x_3x_1) = 2, \quad x_1x_2x_3 = 3.$$

So

$$\begin{aligned} \frac{x_3}{x_1 x_2} + \frac{x_1}{x_2 x_3} + \frac{x_2}{x_1 x_3} &= \frac{1}{x_1 x_2 x_3} (x_1^2 + x_2^2 + x_3^2) \\ &= \frac{1}{x_1 x_2 x_3} [(x_1 + x_2 + x_3)^2 - 2(x_1x_2 + x_2x_3 + x_1x_3)] \\ &= \frac{1}{3} (4^2 - 2 \cdot 2) = 4. \end{aligned}$$

□

Source: Suggested by Dr. E. Wingler.