

Problem # 309

For $0 \leq x \leq 1$ find the maximum value of $\left[\log_{10} \left(\frac{99,999x + 1}{1000} \right) \right]^2$.

Solution:

Answer: The maximum is **9** when $x = 0$.

Proof. Consider $Q = \log_{10} \left(\frac{99,999x + 1}{1000} \right)$ which is monotonically increasing on $0 \leq x \leq 1$. Since Q increases from -3 at $x = 0$ to 2 at $x = 1$ and is 0 when $x = \frac{111}{11,111}$, Q^2 has an absolute maximum of 9 at $x = 0$. □

Source: All the Best from the Australian Mathematics Competition.