

### Problem # 308

Randomly choose three numbers  $p$ ,  $q$  and  $r$  from the set  $\{1, 2, 3, 4, 5, 6, 7\}$ , with replacement so numbers may be repeated. Find the probability that  $pq + r$  is even.

**Solution:**

Answer:  $\boxed{\frac{163}{343}}$

*Proof.* The probability of  $pq$  being odd is  $\left(\frac{4}{7}\right)^2 = \frac{16}{49}$ , so the probability of  $pq$  being even is  $1 - \frac{16}{49} = \frac{33}{49}$ . The probability of  $r$  being odd is  $\frac{4}{7}$ .

The sum  $pq + r$  is even only if

(i)  $pq$  and  $r$  are both odd which has the probability  $\frac{16}{49} \cdot \frac{4}{7} = \frac{64}{343}$ ,

or

(ii)  $pq$  and  $r$  are both even which has the probability  $\frac{33}{49} \cdot \frac{3}{7} = \frac{99}{343}$ .

Thus, the probability that  $pq + r$  is even is  $\frac{64}{343} + \frac{99}{343} = \frac{163}{343}$ .

□

Source: Modified from American High School Mathematics Exam (1995).