

Problem # 168

A positive integer n has a proper divisor d , if d divides n and $1 < d < n$. Find all positive integers n , whose largest proper divisor is 45 times bigger than its smallest proper divisor.

Solution:

Answer: 180 and 405

Proof.

If a is the smallest proper divisor of n , then its largest proper divisor is $\frac{n}{a}$. We know that $\frac{n}{a} = 45a$, that is, $n = 45a^2$. It follows that 3 divides n and thus $a = 2$ or $a = 3$. Therefore, either $n = 45 \cdot 2^2 = 180$ or $n = 45 \cdot 3^2 = 405$.

□

Source: Tournament of Cities, Moscow, 2004.