

Problem # 167

Let $a \neq b$ be real numbers such that $\frac{5a}{a+b} + \frac{5b}{a-b} = 7$. Show that at least one of these numbers is irrational.

Solution:

Proof.

Multiplying both sides of the given equation by $(a-b)(a+b)$ and simplifying, we get $a^2 = 6b^2$. Thus, $\frac{a}{b} = \pm\sqrt{6}$, which proves that at least one of them must be irrational.

□

Source: High School Graduation Exam, Warsaw, Poland, 2007.