

### Problem # 163

Two diagonals  $AC$  and  $BD$  intersect at point  $O$  and cut quadrilateral  $ABCD$  into four triangles:  $\triangle AOD$ ,  $\triangle AOB$  and  $\triangle BOC$  having the areas 30, 75 and 45 respectively. Find the area of the fourth triangle  $\triangle COD$ .

**Solution:**

*Answer:* 18

*Proof.*

Since the triangles  $\triangle AOD$  and  $\triangle AOB$  have the common height, it follows that

$$\frac{|DO|}{|OB|} = \frac{|\triangle AOD|}{|\triangle AOB|} = \frac{30}{75} = \frac{2}{5}.$$

Similarly, 
$$\frac{|\triangle COD|}{|\triangle BOC|} = \frac{|\triangle COD|}{45} = \frac{|DO|}{|OB|} = \frac{2}{5}.$$

It follows that  $|\triangle COD| = \frac{2}{5} \cdot 45 = 18.$

□

Source: Polish Mathematical Forum, 2013.