Problem # 162

Suppose a and b are positive numbers such that $\log_9 a = \log_{15} b = \log_{25}(a + 2b)$. Find the value of b/a expressed in a form that does not involve logarithms.

Solution:

Answer: $\boxed{1 + \sqrt{2}}$

Proof.

Let the common value of the three logarithms be λ. Then $3^{2\lambda} = a$, $3^{\lambda} \cdot 5^{\lambda} = b$ and $5^{2\lambda} = a + 2b$. It follows that $\dfrac{b^2}{a} = a + 2b$ or $\left(\dfrac{b}{a}\right)^2 = 1 + 2\dfrac{b}{a}$. If $t = \dfrac{b}{a}$, then $t^2 - 2t - 1 = 0$ which gives $t = 1 \pm \sqrt{2}$. Since a and b are positive, we have $\dfrac{b}{a} = 1 + \sqrt{2}$.

□

Source: Mu Alpha Theta (1992).