Problem # 160

Find the product of the real roots of the equation,

$$x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}\,.$$

Solution:

Answer: $\boxed{20}$

Proof.

Let $y = x^2 + 18x + 30$. Then the given equation takes the form, $y = 2\sqrt{y + 15}$, from which it follows that $y > 0$. Squaring and rearranging the terms, we get $(y - 10)(y + 6) = 0$, and hence $y = 10$. Going back to the variables x, we obtain $x^2 + 18x + 20 = 0$. This equation has two real roots, whose product is 20.

□

Source: American Invitational Mathematics Examination (AIME) 1983.