

### Problem # 159

Show that for every integer  $n \geq 2$ ,  $(x - 1)^2$  is a factor of  $x^n - n(x - 1) - 1$ .

#### Solution:

*Proof.*

We use the fact that  $(x - a)^k$  is a factor of a polynomial  $f(x)$  if and only if  $f(a) = f'(a) = f''(a) = \dots = f^{(k-1)}(a) = 0$ . Using this with  $a = 1$ ,  $k = 2$  and  $f(x) = x^n - n(x - 1) - 1$ , we check that  $f(1) = 0$  and  $f'(1) = 0$ .

□

Source: Iowa Collegiate Mathematics Competition, 2002.