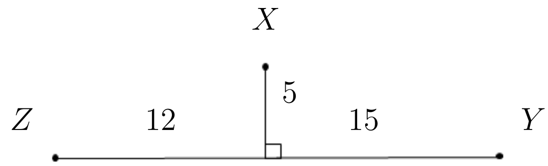
Problem # 158

A circle intercepts the points X, Y and Z shown below. Find the radius of the circle.

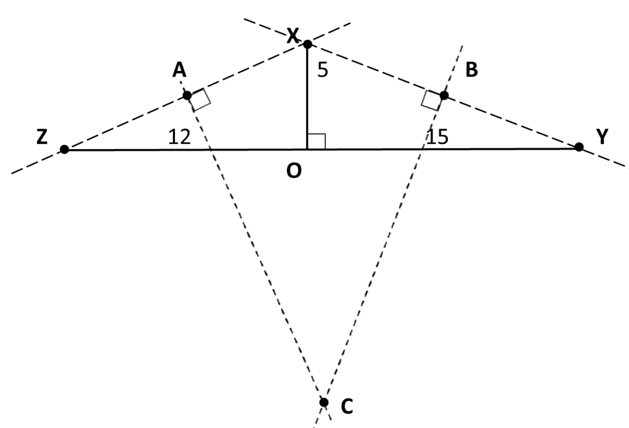


Solution:

Answer: $\boxed{\dfrac{13\sqrt{10}}{2}}$

Proof.

Fix the cartesian coordinates with origin at point O as shown below. The coordinates of X, Y and Z are $(0, 5)$, $(15, 0)$ and $(-12, 0)$ respectively. The midpoints of the chords XY and ZX are $B\left(\frac{15}{2}, \frac{5}{2}\right)$ and $A\left(-6, \frac{5}{2}\right)$ respectively. The perpendicular bisectors of XY and ZX intersect at the center of the circle and their equations are $y = \frac{5}{2} + 3\left(x - \frac{15}{2}\right)$ and $y = \frac{5}{2} - \frac{12}{5}(x + 6)$ respectively. Thus, the center is at point $C\left(\frac{3}{2}, \frac{-31}{2}\right)$, and the radius of the circle is $|CX| = |CY| = |CZ| = \frac{13\sqrt{10}}{2}$.



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Source: Suggested by George Lancaster.

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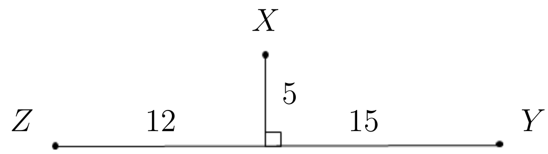
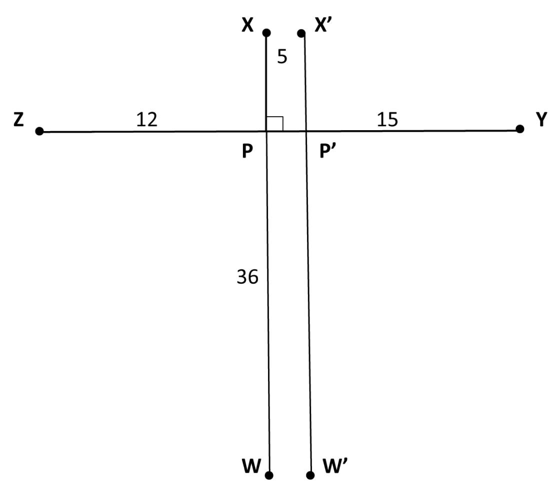


Solution:

Answer: $\boxed{\dfrac{13\sqrt{10}}{2}}$

Proof.

Let r denote the radius of the circle. Extend XP to a point W on the circle. Using the Intersecting Chords Theorem $ZP \cdot PY = XP \cdot PW$ we obtain $PW = 36$. If we reflect all points about the perpendicular bisector of segment ZY, we obtain points X' and W' on the circle and P' on ZY with $WW' = 3$, $X'P' = 5$ and $P'W' = 36$. Observe that $X'W$ is a diameter of the circle and is the hypotenuse of the right triangle $\triangle X'W'W$. By Pythagorean Theorem we obtain $4r^2 = 3^2 + 41^2$ so that $r = 13\sqrt{10}/2$.



Source: Suggested by George Lancaster. □