

Problem # 157

Solve in real numbers x and y the system, or show that it has no solutions:

$$\begin{aligned}x(y - x) &= 3 \\y(4y - 3x) &= 2\end{aligned}$$

Solution:

Answer: System has no solution.

Proof by Thomas Fowler.

Subtracting the first equation from the second gives,

$$4y^2 - 4xy + x^2 = -1 ,$$

which can be factored as

$$(-2y + x)^2 = -1 .$$

By this we see that the solutions to this scenario will be complex, and therefore there will be no real number solutions for x and y .

□

Alternate Proof 1.

Note that $x = 0$ is not a solutions to the system. For $x \neq 0$, the first equation gives $y = x + 3/x$. Substituting this into the second equation and simplifying, we find $x^4 + 13x^2 + 36 = 0$. Solving the quadratic for x^2 , we have the two solutions $x^2 = -9$ and -4 . Since the only solutions for x are complex ($x = \pm 3i, \pm 2i$), the system has no real solutions.

□

Alternate Proof 2.

Let $y = tx$. Then the system takes the form, $x^2(t - 1) = 3$, $x^2(4t^2 - 3t) = 2$. The first equation implies that $t > 1$, and eliminating x we get $12t^2 - 11t + 2 = 0$ which has no solution greater than 1. Thus, the system has no solution.

□

Source: Mathematical Contest, Bialystok Technical University, 2003.